On Harmonic Mean of Certain Triangular Line Graphs

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Abstract—We investigate the harmonic mean of several standard line graphs such as path, Triangular line snakes graphs, Quadrilateral line snakes graphs, etc.

Key words: Quadrilateral Lien Snakes Graphs, Triangular Line Snakes Graphs

I. INTRODUCTION

In this paper we consider simple graphs. Let G= (V, E) be a graph with p vertices and q edges. For all terminologies and notations we follow Harary [1]. There are several types of labeling and detailed survey can be found in. The concept of mean labeling has been introduced in [1] and the Harmonic mean labeling was introduced in [4]. The concept of Double Triangular snake and Double Quadrilateral snake has been proved in [5] and [6]. In this paper we prove that the Harmonic mean behaviour of Triangular line snake graphs, Quadrilateral snake line graphs. Also we wish to investigate Harmonic mean of such graphs.

The following definitions are necessary for our present investigation

A. Definition 1.1:

A graph G= (V,E) with p vertices and q edges is called Harmonic mean graph if its possible to label the vertices x \in V with distant lables from 1,2,3,...,q+1 in such that when

\[ f(e=uv) = \frac{2f(u)f(v)}{f(u)+f(v)} \]

edge e=uv is labeled with then the edge labels of distinct, then “f” is called Harmonic mean labelling in Graph G.

B. Definition 1.2:

Line graphs of a triangular snake graphs

The line graph L(G) of an undirected graph G is another graph L(G) that represent the adjacent between edges of G. In other words given graph G, its line graph L(G) is a graph such that

1) Each vertex of L(G) represents an edge of G.
2) The vertices of L(G) are adjacent iff, there corresponding edges are adjacent to G.

C. Definition 1.3:

A Triangular snake Tn is obtained from a path u1u2.....un by joining ui and ui+1 to a new vertex vi for 1\leq i\leq n-1. That is, every edge of a path is replaced by a triangles C3

D. Definition 1.4:

A Quadrilateral snake Qn is obtained from a path u1u2.....un by joining ui and ui+1 to new vertices vi, wi respectively and then joining vi and wi, 1\leq i\leq n-1. That is, every edge of a path is replaced by cycle C4.

II. MAIN RESULTS

Theorem 1.1: Triangular snakes are Harmonic graphs.

Proof : Consider the following two conditions
1) Case 1 If the triangular starts from the vertex u2, then we define a function f: V (Tn) \rightarrow {1, 2 ,3,...,q+1}. Let us consider the function f(u_{i}, u_{i+1}) = 2i - 1, for all q = 1, 2,3,...(n - 1). f(u_{i}, v_{i}) = (2i - 2) for all i = 2, 4,6,8........(n - 2). f(v_{i}, u_{i+1}) = 2 if or all i = 2, 4, 6,........
(n - 2).

Therefore f is Harmonic mean labelling.

2) Case 2 If the triangular starts from u1, let us define a function f: V (T_{n}) \rightarrow {1, 2,3,........q+1} By this way we can f(u_{i}) = 2i - 1, for all i = 1, 2,3....n . f (u_{i}, v_{i}) = 2i - 1, for i = 1,3,5........ (n - 1), f(v_{i}, v_{i+1}) = 2i + 1, for all i = 1,3,5,...(n-1).

From case 1 and case 2.

Hence the proof.

III. DEFINITION OF QUADRILATERAL SNAKE GRAPHS

Let us consider the path u_{1}, u_{2}, u_{3},........u_{n} by using u_{i}, u_{i+1} to new vertices v_{i}, w_{i} respectively and the joining these vertices which gives a quadrilateral snake graph.

A. Theorem 1.2:

Quadrilateral snake graph Qn is a Harmonic mean on such graphs.

Proof: Let us consider the quadrilateral snake graphs of line graphs Qn.

1) Case 1: Let us consider the quadrilateral snake graphs from u2.

Let us define the new function such that f(u_{1}) = 1, f(u_{2}) = 2, f(u_{3}) = 5, f(u_{i}) = f(u_{i-2}) for all i = 4,5,........n-1). f(v_{i}) = 3, f(v_{2}) = 4, f(v_{i}) = f(v_{i-2}) + 5 for all i = 3,4,5,........(n-1).

Then labeling of each edge of graph as f(u_{1}, u_{2}) = f(u_{1}, u_{3})...........f(u_{1}, v_{i}) = f(u_{1}, v_{i-2}) + 5 for all i = 3,4,5,..........n.

Therefore f is harmonic labeling.

2) Case 2: Starts from u1

f: v (Q_{n}) \rightarrow {1,2,3,........q+1} By f (u_{1}) = 1, f(u_{2}) = 3, f(u_{3}) = 4.

f(u_{i}) = f(u_{i-2}) + 1, i = 3,4,5,........n.

From Case 1 and Case 2, therefore f is Harmonic.

REFERENCES


